MATH 129
FINAL EXAM REVIEW PACKET ANSWERS
(2018)

1. $\int_0^3 \left(10\pi\sin\left(\frac{\pi}{3}t\right)+30\right)dt = 150$ people

2. $\int_1^2 f(5-2x)dx = \frac{7}{2}$ Let $u = 5-2x$ and change the endpoints.

3. a) $\int \frac{t}{\sqrt{t+1}}dt = \frac{2}{3}(t+1)^{3/2} - 2(t+1)^{1/2} + c$ By the method of substitution with $u = t+1$. You can also use integration by parts with $u = t$ and $v' = (t+1)^{-1/2}$. The result is equivalent, just written in a different form. $\int \frac{t}{\sqrt{t+1}}dt = 2t(t+1)^{1/2} - \frac{4}{3}(t+1)^{3/2} + c$

b) $\int \left(\frac{1}{z^2}+A\right)^2 dz = -\frac{1}{3z^3} - \frac{2A}{z} + A^2 z + c$ Distribute.

c) $\int 3^x e^x dx = \frac{1}{\ln(3e)}(3e)^x + c$ Rewrite $3^x e^x = (3e)^x$.

d) $\int_0^1 \frac{\arctan y}{1+y^2}dy = \frac{\pi^2}{32}$ Substitution and change endpoints. Always simplify answer.

4. a) $\int \frac{\ln\left(z^2+1\right)}{z^2}dz = -\frac{1}{z}\ln\left(z^2+1\right) + 2\arctan z + c$ Let $u = \ln(z^2+1)$ and $v' = \frac{1}{z^2}$.

b) $\int x\arcsin(x^2)dx = \frac{1}{2}x^2\arcsin(x^2) + \frac{1}{2}\sqrt{1-x^4} + c$ First make a substitution with $w = x^2$. Then let $u = \arcsin(w)$ and $v' = 1$.

c) $\int_0^1 x\cdot g''(x)dx = 3$ Let $u = x$ and $v' = g''$ for the first integration by parts.

5. a) $\int \cos^2(3\theta+2)d\theta = \frac{1}{6}\cos(3\theta+2)\sin(3\theta+2) + \frac{1}{6}(3\theta+2) + c$ Let $u = 3\theta+2$ before using table formula # 18. If you use another approach, your answer will look different.

b) $\int \frac{2}{4t^2-9}dt = \frac{1}{6}\left(\ln|2t-3| - \ln|2t+3|\right) + c$ Let $u = 2t$ and factor the denominator before using table formula # 26. If you use another approach, your answer will look different.

c) $\int \frac{dy}{\sqrt{y^2+8y+15}} = \ln\left|(y+4) + \sqrt{y^2+8y+15}\right| + c$ Complete the square before using table formula # 29.

d) $\int \frac{\sin(4\alpha)}{\cos^2(4\alpha) - \cos(4\alpha)}d\alpha = \frac{1}{4}\left(\ln|\cos(4\alpha)| - \ln|\cos(4\alpha) - 1|\right) + c$ Let $u = \cos(4\alpha)$ and factor the denominator before using table formula # 26.

6. a) $\int \frac{3y^3+5y-1}{y^3+y}dy = 3y - \ln|y| + \frac{1}{2}\ln|y^2+1| + 2\arctan(y) + c$ First do long division, then use partial fractions $\frac{A}{y} + \frac{By+C}{y^2+1}$.

b) $\int \frac{5z-28}{6z^2+z-40}\,dz = \frac{4}{3}\ln|3z+8| - \frac{1}{2}\ln|2z-5| + c$ Use partial fractions $\frac{A}{3z+8} + \frac{B}{2z-5}$.

c) $\int \frac{dx}{(5-x^2)^{3/2}} = \frac{1}{5}\frac{x}{\sqrt{5-x^2}} + c$ Let $x = \sqrt{5}\sin(\theta)$.

d) $\int \frac{dt}{t^2\sqrt{1+t^2}} = -\frac{\sqrt{1+t^2}}{t} + c$ Let $t = \tan(\theta)$.

7. a) $\int_1^3 f'(x)e^{f(x)}dx = e^{11} - e^7$ Let $u = f(x)$.

b) $\int_1^e \frac{f'(\ln x)}{x}dx = 2$ Let $u = \ln x$.

9. $E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} xf(x)dx + \int_0^{\infty} xf(x)dx = 0 + \int_0^{\infty} x\frac{1}{7}e^{-x/7}dx = 7$
Use integration by parts or the table of integrals. Remember to use proper notation.

10. $\int_0^1 e^{-t^2}dt \approx \frac{1}{2}e^{-1/16} + \frac{1}{2}e^{-9/16} \approx 0.75459794$

11. a), d), and e)

12. Trap, Right, Mid, and Left

13. a) The integral converges. $\int_0^\infty \frac{1}{x^2+4}dx = \frac{\pi}{4}$ Use table formula # 24.

b) The integral converges. $\int_1^\infty \frac{1}{2^x}dx = \frac{1}{2\ln 2}$.

c) The integral diverges. $\int_0^1 \frac{e^x}{\left(e^x-1\right)^2}dx = \infty$ Let $u = e^x - 1$.

d) The integral converges. $\int_{\pi/6}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}}dx = 2\sqrt{\frac{\sqrt{3}}{2}}$ Let $u = \cos x$

e) The integral diverges. $\int_1^\infty \frac{dx}{\left(x-2\right)^3} = \int_1^2 \frac{dx}{\left(x-2\right)^3} + \int_2^\infty \frac{dx}{\left(x-2\right)^3}$. The first integral diverges.

f) The integral converges. $\int_5^\infty \frac{du}{u^2-16} = -\frac{1}{8}\ln\left(\frac{1}{9}\right) = \frac{1}{8}\ln(9)$ Use table formula # 26.

14. $\int_m^\infty e^{-\left(\frac{x-m}{s}\right)^2}dx = \frac{s\sqrt{\pi}}{2}$ Let $u = \frac{x-m}{s}$ and change the endpoints.

15. a) The integral converges. Rewrite as $\int_0^\infty a \cdot f(x)dx = a\int_0^\infty f(x)dx$.

b) The integral converges. Let $u = ax$.

c) The integral diverges. Rewrite as $\int_0^\infty \left(a + f(x)\right)dx = \int_0^\infty adx + \int_0^\infty f(x)dx$.

d) The integral converges. Let $u = a + x$.

16. a) The integral converges. By comparison with $\int_2^\infty \frac{d\theta}{\theta^{3/2}}$.

b) The integral converges. By comparison with $\int_1^\infty \frac{1}{(x+3)^3}dx = \int_4^\infty \frac{1}{u^3}du$.

c) The integral diverges. By comparison with $\int_1^\infty \frac{1}{x}dx$

d) The integral diverges. $\lim_{x\to\infty} \frac{x^5}{e^{-x}+1} = \infty$. In order for the improper integral to converge, the integrand must approach 0.

17. b) and c)

18. a) volume of slice $\approx 16\sqrt{36-(6-x)^2}\Delta x$ Using Pythagorean Theorem.

b) volume of solid $\approx \sum 16\sqrt{36-(6-x_i)^2}\Delta x$ Using the notation of the text.

c) volume = $\int_0^6 16\sqrt{36-(6-x)^2}dx$

19. a) $\int_0^1 3\sqrt{x}dx + \int_1^2 (6-3x)dx$ b) $\int_0^3 \left(\frac{6-y}{3} - \frac{y^2}{9}\right)dy$

20. a) $75\pi - \int_0^3 \pi(5e^{-x})^2 dx = \frac{(125+25e^{-6})}{2}\pi$ b) $\int_0^3 \pi\left(5-5e^{-x}\right)^2 dx = \frac{(75+100e^{-3}-25e^{-6})\pi}{2}$

21. a) $\int_0^8 \pi\left(y^{1/3}\right)^2 dy = \frac{96}{5}\pi$ b) $\int_0^8 \pi\left(y^{1/3}+2\right)^2 dy - 32\pi = \frac{336}{5}\pi$

22. $\int_0^2 \left(\pi\left(f(x)\right)^2 - \pi\left(g(x)\right)^2\right)dx + \int_2^7 \left(\pi\left(g(x)\right)^2 - \pi\left(f(x)\right)^2\right)dx$

23. a) $\int_0^\pi (\sin x)^2 dx = \frac{\pi}{2}$ b) $\int_0^\pi \frac{1}{2}\pi\left(\frac{\sin x}{2}\right)^2 dx = \frac{\pi^2}{16}$

24. Left hand rule:

$$10\cdot\pi\left(\frac{26}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{22}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{18}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{12}{2\pi}\right)^2 + 10\cdot\pi\left(\frac{6}{2\pi}\right)^2 = \frac{4160}{\pi}$$ cubic inches

Right hand rule:

$$10\cdot\pi\left(\frac{22}{2\pi}\right)^2+10\cdot\pi\left(\frac{18}{2\pi}\right)^2+10\cdot\pi\left(\frac{12}{2\pi}\right)^2+10\cdot\pi\left(\frac{6}{2\pi}\right)^2+10\cdot\pi\left(\frac{2}{2\pi}\right)^2=\frac{2480}{\pi}\text{ cubic inches}$$

Trapezoid rule: $\frac{3320}{\pi}$ cubic inches Use the average of the left and right hand rules.

25. a) $\int_0^1\left(\pi\left(\sqrt{y}\right)^2-\pi\left(y^3\right)^2\right)dy$ b) $\int_0^\infty \pi\left(\frac{1}{x^2+1}\right)^2 dx$

26. $\int_0^5\left(2+0.5\cos x\right)dx=10+0.5\sin 5$ grams

27. a) $\int_0^8 \delta(x)\cdot 2\pi x dx$ b) $\int_{-8}^8 \delta(x)\cdot 2\sqrt{64-x^2}\,dx$

28. $\int_0^{25}\left(-\frac{8}{5}h+90\right)\pi(10)^2\,dh=175,000\pi$ pounds

29. a) $a_n=\frac{(-1)^n 2n}{(n+2)^2}$ b) $\lim_{n\to\infty} a_n=-\frac{3}{5}$ $\lim_{n\to\infty} b_n=5$

30. a) $\frac{(3/64)\left(1-(1/4)^8\right)}{1-1/4}=\frac{65535}{1048576}$ b) $\frac{3/4}{1-1/4}=1$

31. $P_n=\frac{(0.05)(200)\left(1-0.05^{n-1}\right)}{1-0.05}$ $Q_n=\frac{(200)\left(1-0.05^n\right)}{1-0.05}$ $n=1,2,3,\cdots$

32. a) The series converges. $\int_2^\infty \frac{1}{x(\ln x)^2}dx=\frac{1}{\ln 2}$ Use the method of section 7.7 to evaluate the improper integral.

b) The series diverges. $\int_1^\infty \frac{3x^2+2x}{\sqrt{x^3+x^2+1}}dx=\infty$ Use the method of section 7.7 to evaluate the improper integral.

33. a) The series diverges. $\lim_{n\to\infty} \frac{e(n)^2}{2(n+1)^2} = \frac{e}{2} > 1$

b) The series converges. $\lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{4} < 1$

34. The Ratio Test gives us 1 and does not tell us anything about the convergence or divergence of the series.

36. a) and b)

37. a) False b) True c) False d) True e) False

38. a) The radius of convergence is $R = 3$. The interval of convergence is $(-7, -1)$.
b) The radius of convergence is $R = \infty$. The interval of convergence is $(-\infty, \infty)$.
c) The radius of convergence is $R = 0$. The series only converges for $x = 1$.

39. a) True b) True c) Impossible to determine.

40. $P_2(x) = 4 + \frac{1}{3}(x-1) - \frac{1}{144}(x-1)^2$ $f(2) \approx P_2(2) = \frac{623}{144} \approx 4.3264$

41. The sign of c_0 cannot be determined, $c_1 > 0$, $c_2 < 0$.

42. a) $f(3) = -1$. b) $f'(3) = \frac{1}{2}$. c) $f''(3) = -\frac{1}{6}$.
d) $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{k!}{(2k)!} 3^k (x-1)^k$ Substitute $3x$ into the series, then simplify.

43. $\frac{1}{12}$ Integrate term by term. The result is recognizable as the series for $\frac{x}{1-x}$.

44. $\cos(2\theta) = -\frac{1}{2} - \sqrt{3}\left(\theta - \frac{\pi}{3}\right) + \left(\theta - \frac{\pi}{3}\right)^2 + \frac{2\sqrt{3}}{3}\left(\theta - \frac{\pi}{3}\right)^3 + \cdots$

45. a) $\sin 1$ b) $\ln(1.5)$ c) Series diverges because $\frac{\pi}{e} > 1$.

46. $-\frac{1}{11}$ Use the series for $\sin x$ to find the series for $\frac{\sin x}{x}$, then consider the term containing x^{11}.

47. a) $x\ln(1+2x) = 2x^2 - \frac{4x^3}{2} + \frac{8x^4}{3} - \frac{16x^5}{4} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{n+2}}{n+1}$

b) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

48. $\frac{a}{(a+r)^2} = \frac{1}{a}\left(1+\frac{r}{a}\right)^{-2} = \frac{1}{a}\left(1-2\left(\frac{r}{a}\right)+3\left(\frac{r}{a}\right)^2-4\left(\frac{r}{a}\right)^3+\cdots\right)$

49. a) False b) False c) False

50. $f(x) = \int_0^x \tan^{-1}(t)dt = \int_0^x \left(t - \frac{t^3}{3} + \frac{t^5}{5} - \frac{t^7}{7} + \cdots\right)dt = \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{30} - \frac{x^8}{56} + \cdots$

The Taylor series for $\tan^{-1}(t)$ at $t = 0$ would be given.

51. $Q = 0$ stable, $Q = 1$ unstable

53. a) *ii* b) *iii* c) *i* d) *iv*

54. $y(t) = e^{3/4}e^{-(1/4)t}$ or $y(t) = e$

55. a) $y(x) = \frac{1}{2}x\sqrt{4-x^2} + 2\arcsin\left(\frac{x}{2}\right) - 1 - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$

b) $x(\theta) = \frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2} + 1 - \frac{\pi}{2}$

c) $y(t) = -\frac{3}{2}e^{4t^2} + \frac{1}{2}$ Watch for the sign issues when you remove the absolute values.

d) $y(x) = 2\sin\left(x + \frac{\pi}{6}\right)$

56. a) $\frac{dQ}{dt} = -\alpha Q$ where $\alpha > 0$ b) $Q(t) = Ae^{-\alpha t}$ c) $t = \frac{7\ln(90)}{\ln(9/5)} \approx 53.59$ hours

57. a) *ii*   b) *iii*   c) *i*   d) *iv*   e) *vi*   f) *v*

58. a) $\frac{dL}{dt} = 4 - 0.6L$, $L(t) = \frac{Ae^{-0.6t} + 4}{0.6}$

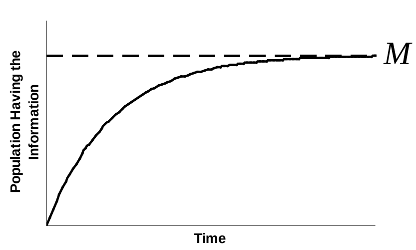
b) The stable equilibrium solution is $L = \frac{20}{3}$. If we start with 20/3 grams per square centimeter of leaves, we will always have that amount.

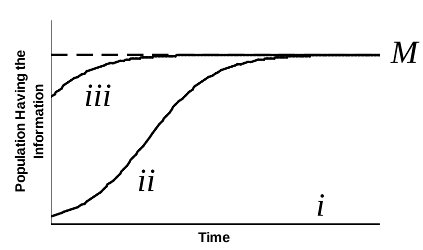
59. a) *i*   b) *ii*   c) *iii*   d) *iv*

60. a) $r(t) = \frac{1}{15}t + 60$ b) $\frac{dH}{dt} = k\left(H - \left(\frac{1}{15}t + 60\right)\right)$ $H(0) = 180$ where $k < 0$

61. $\frac{dA}{dt} = 0.06\sqrt{A}$, $A(t) = (0.03t + c)^2$

62. a) $\frac{dP}{dt} = k(M - P)$ where $k > 0$ b) $\frac{dP}{dt} = kP(M - P)$ where $k > 0$





FOR CLASSES THAT COVERED PHYSICS (SECTION 8.5)

1. a) $\int_0^{15} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ b) $\int_0^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

c) $\int_0^{10} 62.4(h)\pi\left(\frac{8}{15}h\right)^2 dh$ d) $\int_3^{15} 62.4(h+3)\pi\left(\frac{8}{15}h\right)^2 dh$

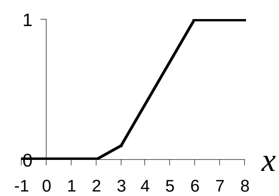
2. $500\cdot 45+\int_0^{45} 3(45-x)dx = 25{,}537.5$ foot-pounds One possibility.

3. $\int_0^{10} 9.8\left(\frac{8}{0.5\cdot 24\cdot 10}\right)(10-h)\left(\frac{12}{5}h\right)dh$ Joules One possibility.

FOR CLASSES THAT COVERED PROBABILITY (SECTIONS 8.7,8.8)

1. a) Probability density function. $c=\frac{1}{10}$.

b) The cumulative distribution function.



2. a) $\int_2^{3} \frac{4}{81}t^3 dt = \frac{65}{81}$ or approximately 80.2%

b) $P(t)=\begin{cases} 0 & t<0 \\ \frac{1}{81}t^4 & 0\le t\le 3 \\ 1 & t>3 \end{cases}$

3. The probability that the time between successive calls is between 5 and 10 minutes:
$F(10)-F(5)=e^{-1}-e^{-2}\approx 0.23$

4. a) $\int_{60}^{\infty} \frac{1}{5\sqrt{2\pi}} e^{-(x-60)^2/50} dx$

b) 0.16 Use symmetry of the graph.